The Immortal Bel–Robinson Tensor

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We present some generalizations, and novel properties, of the Bel–Robinson tensor, in the context of constructing local invariants in D=11 supergravity.

1. Introduction

It is a double pleasure to dedicate this lecture to Luis Bel and to have presented it before both creators of the Bel–Robinson (BR) tensor [1]. Indeed, I feel that I have been one of the prime beneficiaries of this amazing quantity over the years [2], long after they had moved on the better things. What is especially interesting is that many of the applications of BR have been far from the arena of D=4 general relativity for which it was created and intended, and that it has risen from its original incarnation as a would-be tensorial energy-density to its avatar as a basic member of gravitational supersymmetric multiplets and invariants; indeed it is there that it takes on precisely the stress tensor role! Here, I will illustrate the latest twist in the story of BR, namely its essential contribution in the construction of supersymmetric (SUSY) local invariants for D=11 supergravity. This work has been performed jointly with D. Seminara; I refer you to a just-completed compressed version of our results [3], to be followed by a more detailed one. On the BR side, we have been joined by J. Franklin for some computer-based calculations; those results are also to appear in due course [4]. I refer to those papers for details. Here I will only be able to present the general ideas.

Before getting into BR, let me state the reasons why D=11 supergravity and existence of its invariants in particular, are once again important. There are two distinct motivations: (1) if suitable invariants can be constructed, this will imply that it is a nonrenormalizable local field theory, despite its otherwise many attractive properties and (2) the detailed form of these invariants is a clue to the structure of the M-theory that underlies both it and more generally (D=10) string theory as well.

Since this is a mixed audience, I will first provide a mini-resume of the relevant aspects of D=11 supergravity; then we will outline how SUSY invariants can be systematically constructed from this action, and in particular the desired lowest order one, containing scalars quadratic in BR (quartic in curvature). In the remainder of the paper, we will illustrate some of the relevant and useful properties of BR, generalized in three ways: dimension, field content, and tensorial type.

2. D=11 Supergravity

We begin with a brief reminder of this "uniquely unique" highest dimensional supergravity theory [5] whose renewed importance is based on the fact that it is the field theoretical limit of M-theory, the successor as well as generalization of D=10 superstring theory. Recall that D=11 is the highest dimension in which local supersymmetry can be achieved without having to invoke the inconsistent presence of higher spins than 2 and of more than one graviton. Its other claims to uniqueness are threefold: (1) there is only one field content permitted, (2) there can be no "matter" sources, since no lower spin supermultiplets exist, and (3) cosmological constant extension is forbidden [6]. There are simply three fields, the graviton elfbein $e_{\mu a}$, the gravitino ψ_μ and, to balance the number of bose/fermi degrees of freedom, a 3-form potential field $A_{\mu\nu\alpha}$ with totally

antisymmetric field strength $\partial_{[\beta}A_{\mu\nu\alpha]} \equiv F_{\mu\nu\alpha\beta}$, gauge invariant under $\delta A_{\mu\nu\alpha} = \partial_{[\alpha}\xi_{\mu\nu]}$. There are then D(D–3)/2 = 44 gravitons, (D–2)(D–3)(D–4)/3! = 84 form excitations and (D–3)$2^{[D/2]-1} = 128$ gravitinos. The usual SUSY transformation rules

$$\delta e_{\mu a} \sim \bar{\alpha}\Gamma_a \psi_\mu \qquad \delta A_{\mu\nu\alpha} \sim \bar{\alpha}\Gamma_{[\mu\nu}\psi_{\alpha]} \ , \quad \delta\psi_\mu \sim D_\mu \alpha + (\Gamma F)_\mu \alpha \tag{1}$$

leave invariant the action I_{11}, whose leading terms are ($e \equiv \det e_{\mu a}$)

$$I_{11} \sim \int d^{11}x \left[-\tfrac{1}{4}\kappa^{-2} eR + \tfrac{1}{2} e\bar{\psi}(\Gamma D + \Gamma F)\psi - \tfrac{1}{48} eF^2 + 2\kappa/(144)^2 \epsilon^{1..11} F_{1...}F_{5...}A_{..11} \right] . \tag{2}$$

We have dropped all ψ^4 terms; throughout, Γ represents the appropriate member of the D=11 Clifford gamma algebra. Note the last, Chern–Simons (CS), term in its initial physics appearance, and the explicit presence in CS of the Einstein constant, whose dimension is $\kappa \sim [L]^{-9/2}$.

What we are after is apparently both quite simple and far removed from the BR arena, namely the construction of SUSY invariants, like I_{11} itself, under the transformations (1). This turns out to be neither simple nor BR-less! The difficulty lies in the absence of any superfield methods for D=11, so that it is almost impossible to guess the forms of such invariants, or even to verify candidates if they are guessed. This is in contrast with say D=4, where life is much easier and indeed, where BR takes a natural place in the multiplets known there from which invariants are then constructible; as mentioned, BR's role is to replace that of matter stress tensors that underlie lower spin (global SUSY) multiplets.

3. Constructing D=11 Invariants

In this section, we will indicate how it is possible, in a straightforward (though lengthy) way to obtain guaranteed invariants, or more precisely truncations of such invariants to their leading order in $\kappa h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$. We will also (for simplicity) stick with their purely bosonic on-shell components. Let me state the idea: Since I_{11} is an invariant, all scattering amplitudes (at each loop order) it generates also are; since SUSY transformations preserve particle number (also to lowest order) we can just ask for the 4-point tree amplitude with all legs on shell. This object will only be a global SUSY invariant, and of course not fully diffeomorphism invariant – that would require dressing the amplitude with infinitely many external graviton legs as well. But for our physical purposes, which are twofold, that suffices, at least with one more caveat to remove: A free 2 particle→2 particle scattering amplitude is a nonlocal object in general, since it has an intermediate boson propagator (think of matter-matter scattering through an intermediate graviton, $\sim T^{\mu\nu}(p)(p-q)^{-2}T_{\mu\nu}(q)$ in momentum space). To get a local invariant is actually both simple and proves very useful: one just multiplies each term by the product (stu) of the usual Mandelstam variables: one of them knocks out the denominator (s^{-1}, t^{-1} or u^{-1} depending on the channel) while the remaining 4 derivatives (tu, etc.) turn the external graviton and 3-form polarizations into curvatures and field strengths, as required by gauge invariance and explicitly verified by calculation.

The building-blocks in this process are the free particle propagators and cubic vertices in I_{11}, corrected by the 4-point contact terms to preserve gauge invariance. At tree level there is no boson-fermion mixing, *i.e.*, no contribution to the bosonic amplitudes from the fermions. Indeed, it is clear that the relevant bosonic 3-vertices are

$$V_3^g \sim \kappa h \partial h \partial h, \quad V_3^{gF} \sim \kappa h\, FF, \quad V_3^F \sim \kappa \epsilon A F F \tag{3}$$

along with the contact terms $V_4^g \sim \kappa^2 hh\partial h\partial h$ and $V_4^{gF} \sim \kappa^2 hhFF$, required as usual to cancel residual gauge dependence in the 3-vertex contributions. The pure h terms come from expanding the curvature scalar in h, the mixed hFF ones from the form's kinetic part and finally the pure A^3 comes from CS. The vertices in turn give rise to 4-point localized amplitudes of the schematic form:

$$M_4^g \sim R^4\ , \quad M_4^F \sim (\partial F)^4\ , \quad M_4^{g^2F^2} \sim R^2(\partial F)^2\ , \quad M_4^{gF^3} \sim R(\partial F)^3 \tag{4}$$

where we have already multiplied by stu to get local forms and the bosonic part of the total 4-point function (that also contains 4-fermion and mixed 2 fermi-2 boson contributions) is the sum of the terms in (4) and guaranteed to be SUSY invariant, as explained earlier. The respective scatterings described by these M's are: 2 graviton-2 graviton, 2 form-2 form, "graviton Compton" off a form and finally gravitational "bremsstrahlung" from one of the A's in the 3-point CS vertex (the CS vertex itself is of course topological, *i.e.*, gravity-independent). Now as soon as one sees quadratic (let alone quartic) curvature terms, one ought to think of BR. The very good reason for this is of course our experience with lower-dimensional SUSY, most spectacularly in the form of the R^4-like SUSY invariants in D=4 that was used to establish the (3-loop) nonrenormalizability of supergravity there [7]. In that case, it was possible to guess the shape of the invariants by analogy with "squaring" matter supermultiplets which always included $T_{\mu\nu}$; the correct guess was essentially to replace $T_{\mu\nu}$ by $B_{\mu\nu\alpha\beta}$ and hence to expect to have 4-point SUSY invariants that began quartic in the curvature, namely $\sim B^2_{\mu\nu\alpha\beta}$. But at D=11, there is on the one hand no matter "crutch" and on the other, an embarrassment of riches as regards BR; indeed we will see that there is a 3-parameter condidate family to replace the unique totally symmetric traceless conserved BR we know and love in D=4. There is also another complication here: unlike the N=1 in D=4, one has the form field in addition to the graviton (this is more like so-called $N > 1$ models [8]); the ambition now becomes to have a "master" BR that includes the F as well so as to have a simple elegant form to the overall invariant. Fortunately, we have found some general theorems about gravitational couplings that tell us BR will remain a basic "current" also in the present context. Now recall our two aims in this investigation. The first was to decide on whether D=11 supergravity is nonrenormalizable or finite (since κ^2 is dimensional, it is one or the other – it cannot be renormalizable), by exhibiting a local candidate counterterm or showing none exists. For this purpose, it is the invariant's existence, not the details or elegance of its form, that counts.[1] But there is another aim that is really more fundamental, namely to use the D=11 model as a probe of the as yet unknown underlying M-theory. For, just as the nonlocal D=10 superstring theory reduces to D=10 supergravity in the zero-slope limit, but further induces (an infinite series of) corrections, so should M-theory reduce to D=11. As an amusing sidelight, these "zero slope" corrections in the graviton-graviton sector turn out to have the same form in both D=10 and D=11; they are in turn equivalent to the "localized" tree-level pure Einstein 4-graviton scattering amplitude, which can be written as [10] $t_8 t_8 RRRR$ where t_8 is effectively an 8-index constant tensor made out of Lorentz metrics. The relation with $(\mathrm{BR})^2$ can then be obtained by expanding both in a basis of quartic invariants [11]. Because of the numerous probes of M-theory that are being undertaken using brane dynamics, it is paramount to have exact forms and relative coefficients of the corrections (our invariant being the first such) to be matched against brane calculations. Clearly, any underlying unified symmetry (or even good "notation" such as BR!) to be found in the invariant would be very important for this purpose. Rather than give details of our procedure or its results [3], I move now to our main subject of BR,

[1] Establishing the nonvanishing of its coefficient at the relevant loop order where it can appear requires a separate calculation. For the present model, this was essentially carried out in [9].

its properties in arbitrary dimensions, its link to other invariants, and also to a solution of an old problem in [12] attempting to relate BR to gravitational pseudotensors.

As a concluding remark in this section, it may be helpful to mention an aspect of SUSY and of supergravity that sometimes confuses classical relativists. Let us phrase it as a question: why should purely gravitational quantities (here tree level graviton scattering amplitudes) that also happen to arise in supergravity display any "SUSY" properties, particularly since no fermions at all are involved at tree level. The point is that the *supersymmetrizability* that is inherent in the Einstein action (but *not*, say, in its cosmological extension in D=11 as we have mentioned) already provides strong *a priori* constraints on its internal properties, quite apart from whether we choose to implement the SUSY extension. Historically, for example, the positive energy theorem was obtained this way, as was the simple structure of the D=4 graviton-graviton scattering amplitude, the result of an otherwise horribly complicated purely gravitational calculation [10]. The simplicity follows from an underlying helicity conservation, which is a spinoff inherent to supersymmetrizability [13].

4. BR

Let's first pay homage to the original, D=4, definition [1],

$$B_{\mu\nu\alpha\beta} = R^{\sigma}{}_{\mu\tau\alpha}\, R_{\sigma\nu}{}^{\tau}{}_{\beta} + {}^{*}\!R^{*\sigma}{}_{\mu\tau\alpha}\, {}^{*}R_{\sigma\nu}{}^{\tau}{}_{\beta}\ , \quad {}^{*}R^{\mu\nu}{}_{\alpha\beta} \equiv \tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma} R_{\lambda\sigma\alpha\beta}\ . \tag{5}$$

This tensor is fully symmetric, traceless, covariantly conserved on shell (*i.e.*, in Ricci-flat geometries), vanishes *iff* Riemann does and even has positive energy density B_{0000}, just like its model, the Maxwell stress tensor. On the other hand, charges made from B don't really exist (last paper in [2]); it is a "zilch" as in fact was (indirectly) established much earlier: adding Ricci-dependent terms converts BR to an identically conserved quantity [14], hence without dynamical content. However, this in no way diminishes the importance of BR, in particular in the supergravity arena.

First, while still in D=4, we exhibit the promised solution of the MTW problem: can one simply relate BR to the energy (necessarily *pseudo*-) tensors $t_{\mu\nu}$ of gravity? Apart from the amusing aspect of the question, there is actually a deep point that is essential to the generalization of BR to include matter and our form fields as well as gravity. Basically, just by dimensions, BR has two more derivatives (and indices) than does any $t_{\mu\nu}$. One therefore expects any such relation to be of the schematic form $B_{\mu\nu\alpha\beta} \sim \partial^2_{\alpha\beta} t_{\mu\nu}$, though of course it could only hold in some gauge since neither t nor its derivatives are tensors. The obvious gauge is that of Riemann normal coordinates (RNC) at a point so that all the affinities $\Gamma^{\lambda}_{\rho\sigma}$ vanish for simplicity, and we don't have to worry about "spreading out" the $\partial^2_{\alpha\beta}$ over the $\Gamma\Gamma$ of $t_{\mu\nu}$. It turns out, surprisingly, that such a relation actually holds without any remainder (unlike the one in [12]); I omit the gory details [4]. The result is that, in RNC,

$$B_{\mu\nu\alpha\beta} = \partial^2_{\alpha\beta}(t^{LL}_{\mu\nu} + \tfrac{1}{2}\, t^{E}_{\mu\nu}) + 0\ . \tag{6}$$

in terms of the Landau–Lifschitz and Einstein pseudo-tensors, in a standard normalization.

Some of the simplicity of B in D=4 is, alas, very specific to this (degenerate) dimension, one in which the number of independent quartic curvature invariants drops from 7 to 2. Of particular physical interest in supergravity there is that its (unique) square $B^2_{\mu\nu\alpha\beta}$ can be written in several equivalent ways,

$$2\,B^2 = [(R_{\mu\nu\alpha\beta})^2]^2 - [R_{\mu\nu\alpha\beta}R^{\mu\nu\lambda\sigma}]^2 \sim E_4^2 - P_4^2 \equiv (E_4+P_4)(E_4-P_4) \tag{7}$$

where $(E_4,\ P_4)$ are respectively the Euler (${}^{*}R^{*}R$) and Pontryagin ($R^{*}R$) topological densities of D=4. The first equality is already surprising: a square is also a difference of squares. The second

equality says it is also the difference of two scalars ($E_4^2,\ P_4^2$) that can in fact be chosen to span the quartic invariant basis. The last equality, while obvious, expresses the totally helicity conserving (or violating, depending on in/out conventions) character of the 4-graviton scattering amplitude in D=4 that I referred to earlier [13]; since it is essentially proportional to B^2. What makes D=4 special is the quadratic identity

$$S_{\mu\nu} \equiv R_{\mu\alpha\beta\gamma} R_\nu{}^{\alpha\beta\gamma} - \tfrac{1}{4} g_{\mu\nu} R^2_{\alpha\beta\gamma\delta} \equiv 0, \quad D=4 \tag{8}$$

which is derivable from the fact that any antisymmetrization over $D+1$ indices vanishes identically at D, *i.e.*, $R^{\mu\nu}_{[\mu\nu} R^{\alpha\beta}_{\alpha\beta} X^\lambda_{\lambda]} \equiv 0$ for any X. Note that the tensor in (8) no longer zero, but is on-shell conserved in any D, thereby providing one "free" family of BR-like tensors there (it is proportional to the trace of (5) in fact). Next, let me turn to the generalizations of $B_{\mu\nu\alpha\beta}$ in arbitrary dimension D. Beyond D=4, there is actually a 3-parameter family of what may legitimately be called the successors of BR, depending on which of its properties one wishes to keep, apart from that of conservation on (at least) two indices, say ($\mu\nu$). Of course, one representative of B in any D is just (5) itself with the $\epsilon\epsilon$ expanded out,

$$B_{\mu\nu\alpha\beta} = R^\sigma{}_\mu{}^\tau{}_\alpha R_{\sigma\nu\tau\beta} + R^\sigma{}_\mu{}^\tau{}_\beta R_{\sigma\nu\tau\alpha} - \tfrac{1}{4} g_{\mu\nu} R_\alpha{}^{\sigma\tau\delta} R_{\beta\sigma\tau\delta}\ . \tag{9}$$

The leading (trace-independent) terms are common to this whole family. Various choices of the parameters in this extended BR will yield specific properties such as tracelessness, total symmetry, or conservation on all indices. As a first step, define the form

$$\overline{B}_{\mu\nu\alpha\beta} = B_{\mu\nu\alpha\beta} - \tfrac{1}{4} g_{\alpha\beta} B_{\mu\nu\rho\sigma} g^{\rho\sigma} \tag{10a}$$

which enjoys the all-index conservation property. In terms of $\overline{B}$, the three-parameter family then reads

$$B^{(a)}_{\mu\nu\alpha\beta} \equiv \overline{B}_{\mu\nu\alpha\beta} + a_1 \overline{B}_{\mu\alpha\nu\beta} + a_2 \overline{B}_{\mu\beta\nu\alpha} + a_3 g_{\alpha\beta} \overline{B}_{\mu\nu\rho\sigma} g^{\rho\sigma}\ . \tag{10b}$$

Again we stress that (only) in D=4, (10b) reduces to the original, unique, form (5).

Coming now to matter extensions, let me first show that they naturally arise within the context of (massless) matter interaction with gravity, even apart from SUSY. Consider, as a pedagogical example, scattering of massless scalars through their gravitational couplings; to lowest order,

$$\begin{aligned} L_{\rm int} &= \kappa h^{\mu\nu}\, T_{\mu\nu}(\phi) \quad T_{\mu\nu}(\phi) \equiv \phi_\mu \phi_\nu - \tfrac{1}{2}\eta_{\mu\nu}\phi^2_\alpha \\ \kappa h_{\mu\nu} &\equiv g_{\mu\nu} - \eta_{\mu\nu}\ , \quad \phi_\mu \equiv \partial_\mu \phi \end{aligned} \tag{11}$$

plus the contact term $\sim hh\partial\phi\partial\phi$ needed to preserve gauge invariance. Then the resulting 4-scalar amplitude due to graviton exchange according to (11), using the graviton propagator $\sim k^{-2}$ and extracting the local part of this by multiplication with stu, is[2]

$$B_{\mu\nu\alpha\beta}(\phi) = \phi_{\alpha\mu}\phi_{\beta\nu} + \phi_\beta \phi_{\alpha\nu} - \eta_{\mu\nu}\phi_{\alpha\sigma}\phi_\beta{}^\sigma \tag{12}$$

where $\phi_{\mu\nu} \equiv \partial^2_{\mu\nu}\phi$. This quantity is separately ($\mu\nu$) and ($\alpha\beta$) symmetric and conserved on ($\mu\nu$) on-shell ($\Box\phi = 0$). It can even be covariantly completed, to become covariantly conserved (on

[2] I have been informed that Teyssandier and Bel (unpublished) constructed similar $B(\phi)$; a Maxwell $B_{\mu\nu\alpha\beta}(F_{\lambda\sigma})$ has been defined in [15], following the $\partial_{\alpha\beta} t_{\mu\nu}$ gravitational idea.

Einstein + scalar shell), by turning all $\partial_\mu \to D_\mu$ and adding to (12) curvature-dependence of the form

$$2\Delta B_{\mu\nu\alpha\beta}(\phi) \sim R_{\mu\sigma\nu\beta}\,\phi^\sigma\phi_\alpha + (\mu\nu) + (\alpha\beta)\ , \tag{13}$$

i.e., adding the 3 terms indicated by the symmetrizations.

So our interest is not so much to play BR for its own sake. Rather it is to see how we are led, within the rather different framework of SUSY invariant scattering amplitudes, not only to this sort of $B_{\mu\nu\alpha\beta}(F_{\gamma\delta\sigma\tau})$ for our form fields, but also to unified

$$B_{tot} \equiv B(g) + B(F) \tag{14}$$

expressions. As might be expected from the universal vertex coupling forms, the $B(F)$ should look very similar. Indeed, $B(F)$ (with again admits of several parameter extensions) has the leading $\partial^2_{\alpha\beta}T_{\mu\nu}(F)$ form, namely

$$B(F)_{\mu\nu\alpha\beta} = \partial_\alpha F_\mu\partial_\beta F_\nu + \partial_\beta F_\mu\partial_\alpha F_\nu - \tfrac{1}{4}\,\eta_{\mu\nu}\partial_\alpha F\partial_\beta F, \quad \partial^\mu B(F)_{\mu\nu\alpha\beta} = 0 \tag{15}$$

where all omitted indices are traced in an obvious way. Its conservation holds on shell, $\partial^\mu F_{\mu\nu\alpha\beta} = 0$, using the Bianchi identities $\partial_{[\lambda}F_{\mu\nu\alpha\beta]} \equiv 0$. One can even go further and incorporate fermionic $B(\psi_\lambda)$ where ψ_λ is the gravitino potential. It has a form (again $\sim \partial^2_{\alpha\beta}T_{\mu\nu}(\psi)$) similar to the gravitational B, and is quadratic in the fermionic curvature $f_{\mu\nu} = \partial_\mu\psi_\nu - \partial_\nu\psi_\mu$, $B(\psi) \sim \kappa^2(\bar{f}\Gamma\partial f)$. Its form is relevant to the full SUSY invariant that includes the 4-fermion and mixed, 2-fermion—2-boson, amplitudes.

5. Conclusions

After 40 years, not only has the original unique D=4 gravitational BR of [1] led us from (5) to the higher complexities (and usefulness!) of expressions such as (14), but also to a different class of "currents" that have proven to be equally essential building-blocks of SUSY invariant expressions. These include gravitational 4-forms

$$P_{\mu\nu\alpha\beta} = \tfrac{1}{4}R^{\lambda\sigma}_{[\mu\nu}R_{\alpha\beta]\lambda\sigma} \tag{16}$$

that are closed rather than conserved; they are of course easily overlooked in D=4 where they reduce essentially to the Pontryagin scalar $P_{\mu\nu\alpha\beta} \to \epsilon_{\mu\nu\alpha\beta}(R^*R)$. Then there are mixed gravity-form conserved currents

$$C^{RF}_{\mu\nu\rho;\alpha\beta} \equiv \partial_\lambda\left[R^\alpha{}_{(\alpha}{}^{[\lambda}{}_{\beta)}F_\sigma{}^{\mu\nu\rho]}\right] - \tfrac{1}{6}R^\sigma{}_{(\alpha}{}^{\lambda}{}_{\beta)}\partial_\lambda F_\sigma{}^{\mu\nu\rho}\ , \tag{17}$$

that involve both bosonic building blocks of D=11 supergravity and the list goes on.

This tribute to BR and its serendipitous importance in supergravities at all dimensions, as well as the many ways in which it generalizes, should assure us that BR industries (like its founders) still have a great future.

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